Too interconnected to fail:
Contagion and Systemic Risk in Financial Networks

Rama CONT

Joint work with:
Amal Moussa (Columbia University)
Andreea Minca (Université de Paris VI)
Edson Bastos (Banco Central do Brasil)
Financial stability and systemic risk

The recent financial crisis has simultaneously underlined the importance of contagion and systemic risk and the lack of adequate indicators for measuring and monitoring them.

Control over systemic risk has been the main motivation of the recent bailouts of large financial institutions.

Regulators have had great difficulties anticipating the impact of defaults partly due to a lack of visibility and lack of relevant indicators on the structure of the financial system.

Policy has been guided by “too big to fail” principle.

Availability of better indicators of systemic risk would have greatly helped regulatory and crisis management policy.
A need for indicators of systemic impact

- US Treasury has called “for new legislation granting additional tools to address systemically significant financial institutions” (Mar 2009).

- The new legislation ” would cover financial institutions that have the potential to pose systemic risks to our economy”.

- “In determining whether to use the program for an institution, Treasury may consider the extent to which destabilization of the institution could threaten the viability of creditors and counterparties exposed to the institution whether directly or indirectly.”

- What makes an institution systemically significant”?

- Need for indicators of systemic impact of the failure of a financial institution
OBJECTIVES

In this work we

- propose a quantitative approach for measuring the systemic impact of the failure of a large financial institution: the Systemic Risk Index

- This index combines the effects of
  - common market factors affecting defaults
  - default contagion via counterparty risk
  - indirect contagion via credit default swaps

- use this measure of systemic risk on empirical data and simulated network structures to study the influence of network structure, credit default swaps and clearinghouses on systemic risk in the banking system.
Systemic vs marginal risk

- Bank regulation has focused on the risk of individual financial institutions (VaR) to determine capital requirements.
- Capital should be sufficient to cover typical losses of large magnitude.
- VaR of an institution measures how much the institution can be harmed by market moves: it is concerned with the marginal loss distribution of the institutions portfolio.
- On the contrary, systemic risk is concerned with how much the financial system can be harmed by the failure of the institution.
- It is concerned with the joint distribution of the losses of all market participants and requires modeling how losses are transmitted through the financial system.
**LTCM**

- Amaranth: size = 9.5 billion USD.
- The default of Amaranth hardly made headlines: no systemic impact.
- The default of LTCM threatened the stability of the US banking system → Fed intervention.
- Reason: LTCM had many counterparties in the world banking system, with large liabilities/exposures.
- **Point 1**: Systemic impact/ default contagion is not about the size of a firm.
- **Point 2**: a firm’s portfolio can be “well-hedged” (low market risk using conventional measures) but the firm can be a source of large systemic risk.
The network approach to contagion modeling

We model a network of counterparty relations as a *weighted directed graph* whose

- $n$ vertices (nodes) $i \in V$ represent financial market participants: banks, funds, corporate borrowers and lenders, hedge funds, insurers, monolines.

- (directed) links represent counterparty exposures: $L_{ij}$ is the (market) value of liabilities of $i$ towards $j$, $L_{ji}$ is the exposure of $i$ to $j$.

- In a market-based framework $L_{ji}$ is understood as the fair market value of the exposure of $i$ to $j$.

- Each institution $i$ disposes of a *capital buffer* $c_i$ for absorbing market losses. In practice: Tier I+II capital minus required capital for non-banking assets.
- Solvency condition: \( c_i + \sum_j L_{ji} - \sum_j L_{ij} > 0 \)
- Capital absorbs first losses. Default occurs if \( \text{Loss}(i) > c_i \).

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank assets</td>
<td>Interbank liabilities</td>
</tr>
<tr>
<td>( \sum_j L_{ji} )</td>
<td>( \sum_j L_{ij} )</td>
</tr>
<tr>
<td>Other assets</td>
<td>Capital</td>
</tr>
<tr>
<td>( A_i )</td>
<td>( C_i )</td>
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Table 1: Stylized balance sheet of a bank.
Brazil’s interbank network

Joint work with Edson BASTOS, Banco Central do Brasil.
Data from 2008 provided by Bank of Brazil.

- Complete data set of all consolidated interbank exposures (including swaps) + Tier I and Tier II capital.
- \( n \approx 100 \) institutions (holdings), \( \approx 1000 \) counterparty relations
- Average number of counterparties (degree) = 7
- In-degree (number of debtors) has a Pareto distribution with exponent \( \approx 2 \).
- Out-degree (number of creditors) has a Pareto distribution with exponent \( \approx 3 \).
- Exposures sizes very heterogeneous: heavy tailed distribution, a handful of bilateral exposures are \( > 100 \) times larger than most of the rest.
Figure 1: Brazilian interbank network (Cont & Bastos 2009).
Figure 2: Brazilian interbank network: distribution of degree (number of counterparties).
Figure 3: Brazilian interbank network: distribution of in-degree (number of creditors): Pareto tail with tail index $\simeq 1.7$
Figure 4: Brazilian interbank network: distribution of out-degree (number of debtors): Pareto tail with tail index $\approx 3$
Figure 5: Brazilian interbank network: distribution of clustering coefficient
Figure 6: Brazilian interbank network: degree vs clustering coefficient.
Figure 7: Rank diagram of largest counterparty exposures of AIG in Sept 2008 exhibits a similar Pareto tail.
A preferential attachment model for banking networks

The graph $G(t)$ is constructed iteratively in $t$ steps starting from an initial node. At step $t + 1$

- with probability $1 > p > 0$, add a new vertex $v$ together with an edge from $v$ to an existing vertex $w$ chosen with probability

$$\frac{\text{deg}_{\text{in}}(w) + \delta_{\text{in}}}{\sum_{i=1}^{\lvert G(t) \rvert} (\text{deg}_{\text{in}}(i) + \delta_{\text{in}})}.$$  \hspace{1cm} (1)

- With probability $p$, add a new vertex $w$ and an edge from an existing vertex $v$ to $w$, where $v$ is chosen with probability

$$\frac{\text{deg}_{\text{out}}(w) + \delta_{\text{out}}}{\sum_{i=1}^{\lvert G(t) \rvert} (\text{deg}_{\text{out}}(i) + \delta_{\text{out}})}.$$  \hspace{1cm} (2)

- With probability $1 - 2p$, link an existing vertex $v$ to an existing vertex $w$, where $v$ and $w$ are chosen independently, $v$ with
probability (2) and w with probability (1).

The construction is stopped when \(|G(t)| = N\).

Interpretation: a new firm entering the financial system is more likely to establish financial links with highly connected firms.

Once the graph is constructed we allocate IID exposures (weights) to each link with a Pareto distribution with exponent 2.
Property 1 (Degree distributions). With probability 1, we have

$$\frac{1}{N} \# \{ v \in [G_N], \text{indeg}(v) = k \} \xrightarrow{a.s.} q_{\text{in}}(k)$$  \hspace{1cm} (3)

$$\frac{1}{N} \# \{ v \in [G_N], \text{outdeg}(v) = k \} \xrightarrow{a.s.} q_{\text{out}}(k)$$  \hspace{1cm} (4)

as $N \to \infty$. Furthermore there exists constants $C_{\text{in}}, C_{\text{out}} > 0$ such that

$$q_{\text{in}}(k)^{k \to \infty} \sim \frac{C_{\text{in}}}{k^{\alpha_{\text{in}}}} \quad q_{\text{out}}(k)^{k \to \infty} \sim \frac{C_{\text{out}}}{k^{\alpha_{\text{out}}}}$$  \hspace{1cm} (5)

with

$$\alpha_{\text{in}} = \frac{2 - p + 2p\delta_{\text{in}}}{1 - p} \quad \alpha_{\text{out}} = \frac{2 - p + 2p\delta_{\text{out}}}{1 - p}$$  \hspace{1cm} (6)

In particular choosing $p = 0.1, \delta_{\text{in}} = 0, \delta_{\text{out}} = 4.45$ we obtain

$$\alpha_{\text{in}} = 2.1 \quad \alpha_{\text{out}} = 3.1$$
Figure 8: Degree distributions obtained for random attachment model. In-exponent: 2.1; Out-exponent: 3
Figure 9: Left: distribution of clustering coefficient. Right: clustering coefficient vs degree.
Contagion in banking networks: theory


Theoretical results on the influence of network structure on contagion have been obtained only for a limited number of highly stylized structures of interbank markets, chosen more for analytical convenience than for their resemblance to real world banking systems.

These studies suggest however that the magnitude of contagion depends on
the size of interbank exposures relative to capital
the precise pattern of such linkages (network structure).
Contagion in banking networks: empirical studies

Empirical studies on interbank networks by central banks:

- Sheldon and Maurer (1998) for Switzerland
- Furfine (1999) for the US
- Upper and Worms (2000) for Germany Wells (2002) for the UK
- Boss, Elsinger, Summer and Thurner (2003) for Austria
- Mistrulli (2007): Italy

Examine by simulation the impact of single or multiple defaults on bank solvency in absence of other effects (e.g. market shocks).

Mostly focused on payment systems (FedWire, etc)
The small magnitude of such “domino” effects has been cited as justification for ignoring contagion e.g. in the Geneva Report. Such simulation ignore the impact of correlated market shocks on bank balance sheets.

Except Mistrulli (2007), most other studies use maximum entropy/other reconstruction techniques which result in distributing as uniformly as possible liabilities among counterparties. This method can also lead to underestimation of contagion effects.

“Market clearing equilibrium”: amounts to computing cash flows assuming simultaneous liquidation of all market participants positions. Defaults are then generated endogenously.

Not a realistic situation: defaults are not generated by global market clearing but may appear as exogenous shocks to capital reserves of banks.

Portfolio approach (Lehar 2005, Elsinger et al 05): consider the financial system as a portfolio, simulate its loss distribution and compute a risk measure (Value at Risk) for this “portfolio”.

Such global measures do not isolate the impact of a single default or compute the systemic impact of a given institution.

Also, these studies have ignored the impact of credit risk transfer instruments such as credit default swaps (notional of 30 trillion USD).
Measuring the systemic impact of a default

Objective: quantify the losses generated across the network by the initial default of a given financial institution.

We consider two mechanisms which contribute to contagion in the network:

1. default contagion across counterparty networks
2. correlated, market shocks
Mechanism 1: market-induced credit event

If a bank $i$ faces unexpected market loss (e.g. writedown of assets resulting from large sudden market moves), it can default if the loss exceeds its capital $c_i$

Such losses can arise from

- Exogenous market shocks: this is modeled by applying correlated shocks $\epsilon_i$ across balance sheets

$$c_j \mapsto c_j + \epsilon_j$$

- Margin calls or payments on bilateral contracts to another market participant $j$ ex. credit default swaps triggered by credit events in the network.

$$c_j \mapsto c_j - \Pi_{ij}$$
Mechanism 2: contagion via counterparty risk

The default of a market participant $i$ affects its counterparties in the following way over a short term horizon

- Debts are collected from debtors at liquidation:

  $\forall j, L_{ji} \rightarrow 0$

- Creditors loses a fraction $(1 - R)$ of their exposure.

  Loss is first absorbed by capital: $c_j \rightarrow \min(c_j - (1 - R)L_{ij}, 0)$.

  This leads to a writedown of $(1 - R)L_{ij}$ in the balance sheet of $j$, which can lead to **insolvency** of $j$ if

  $c_j < (1 - R)L_{ij}$

  Typically $R \simeq 0$ in the short term.
**Default Impact of an institution**

Default of a market participant $i$ may incur losses of its counterparties. These losses may lead the counterparties to become in turn insolvent. This may generate a “cascade” of defaults by iterating this procedure.

We define the “default impact” $DI(i)$ of a network node $i$ as the total loss (in $\$) along the cascade of defaults generated by the initial default of $i$.

It is the response of the network to the suppression of node $i$ in absence of other effects.

$DI(i)$ does not involve estimating the default probability of $i$. It is a ‘worst case’ loss estimate.

Similarly we can define/compute the Default Impact $DI(A)$ of a set $A \subset V$ of nodes.
Default contagion during a crisis: combining market risk and contagion effects

Default contagion during a crisis: evaluate impact of a large default in presence of (correlated shocks) to capital structure across institutions

1. Network structure \((c(0), L)\) is given at \(t = 0\)

2. Market risk of bank portfolios is simulated using a factor model: (correlated) downward shocks are applied to balance sheets of different institutions

\[
c_j(T) = c_j(0) + \epsilon_i \quad (\epsilon_i, i \in I) \sim F
\]

3. We now consider default of \(i\) and compute the Default Impact \(DI(i)\) in the “stressed market” environment characterized by \((c(T), L)\)

\(DI(i)\) is now a random variable depending on \(c(T)\).
Ex: Gaussian one-factor model $\epsilon_i(T)$

$$c_i(T) = F_i^{-1}(N(X_i)) \quad X_i = (\sqrt{\rho}Z_0 + \sqrt{1 - \rho}Z_i)$$

where $Z_i$ are IID $N(0,1)$.

Ex: a heavy-tailed factor model $\epsilon_i(T)$

$$c_i(T) = F_i^{-1}(G_i(X_i)) \quad X_i = (\sqrt{\rho}Z_0 + \sqrt{1 - \rho}Z_i)$$

where $Z_i$ are Stable$\alpha$) with $\alpha = 1$.

Lehar (2005) gives estimates for volatilities and correlations of assets of international banks: $\rho \in [0.2, 0.6]$.

More generally one can use other factor model commonly used in portfolio default risk simulations.
**Systemic Risk Index** of a financial institution

We now combine the (deterministic) computation of Default Impact and the (stochastic) simulation of correlated defaults at horizon $T$ and define the *Systemic Risk Index* of the institution $i$ at a horizon $T$ as

$$S(i) = E[DI(i) | c_i(T) \leq 0]$$

It is the expected loss in the cascade generated by the failure of $i$, given that at the time of failure the capital buffer of $i$ has been wiped out by market shocks.

This indicator combines market-based measures of default probability and correlation/dependence with a network-based measure of default contagion.
**Systemic Risk index as a risk measure**

Similarly we can define the Systemic risk contribution of a set $A \subset V$ of financial institutions: it is the expected loss to the financial systems generated by the joint default of all institutions in $A$:

$$S(A) = E[DI(A)|\forall i \in A, c_i(T) \leq 0]$$

$S$ then defines a set function

$$S : \mathcal{P}(V) \mapsto \mathbb{R}$$

The Systemic Risk Index can be viewed, from the point of view of the *regulator*, as a macro-level “risk measure”.
Submodularity of Default Impact and Systemic risk

Consider a function $f : \mathcal{P}(V) \mapsto \mathbb{R}$ defined on the subsets of a set $V$. (e.g. set of nodes of a graph)

**Definition 1** (Submodularity). $f : \mathcal{P}(V) \mapsto \mathbb{R}$ is said to be submodular if for any subsets $A \subset B \subset V$,

$$\forall x \in V, f(\{x\} \cup A) - f(A) \geq f(\{x\} \cup B) - f(A)$$

This property is the discrete analog of *concavity* for continuous functions.
Submodularity of default impact and systemic risk index

Proposition 1.

- **Deterministic case:** The loss function $S: \mathcal{P}(V) \mapsto [0, \infty]$ is increasing and submodular.

- **The Systemic Risk Index** $\bar{S}$ in the combined market stress/contagion model is submodular if the marginal density $F_i$ of the market shocks $\epsilon_i$ at each node has an increasing density on $]-\infty, c_i(0)]$. 

*Ex:* Gaussian, Student or double-exponential shocks centered at $\mu > c_i(0)$ will lead to submodular loss functions.
Simulation results

We generate a directed scale-free network with $n = 400$ nodes with Pareto distributions for degree and exposure sizes which match the empirical properties of Brazilian and Austrian networks.

- Heterogeneity of connectivities: in/ out-degree has Pareto distribution with exponents 2.1 and 3.
- Exposures $L_{ij}$ are IID with a Student distribution with $\nu = 1.9$ degrees of freedom ($\rightarrow$ Pareto tail)

We consider two different situations for the capital:

- Limit on leverage: $c_i \geq \alpha \sum_j L_{ij}$ e.g. $12\% \leq \alpha \leq 5\%$
- Capital proportional to net value of positions (Basel II): allows for large/unlimited leverage

Default impact is computed for each node.

Systemic risk index is computed by Monte Carlo using a variance
reduction (importance sampling) method for efficiently sampling joint default events

To quantify the impact of imposing a maximal leverage ratio without increasing total amount of available capital reserves we conduct an experiment where the ratio is fixed in a way that the total amount of capital reserves summed across institutions is the same in both cases.
Figure 10: ‘Too big to fail”? Systemic risk index vs size.
Figure 11: Influence of leverage regulation on default impact: imposing a cap on leverage reduces contagion.
Figure 12: Influence of leverage regulation on default impact: imposing a cap on leverage reduces proportion of institutions with large default impact.
Figure 13: **Imposing a cap on leverage reduces default impact of systemically important institutions:** Loss generated by the institution with highest default impact as a function of the minimal ratio of capital to total exposures.
Figure 14: Distribution of number of defaults generated in a cascade: imposing a cap on leverage reduces contagion.
Figure 15: Influence of leverage regulation on distribution of default impact: imposing a cap on leverage reduces probability of large systemic losses.
Figure 16: Distribution of systemic risk index: no cap on leverage.
Figure 17: Systemic risk index for 5 most systemic nodes in units of average SRI.
Figure 18: Systemic risk index vs total liabilities: evidence of contagion in Brazil interbank network.
Figure 19: Distribution of systemic risk index: Brazil interbank network.
<table>
<thead>
<tr>
<th>Default impact</th>
<th>Network average</th>
<th>5 most systemic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree</td>
<td>0.2 %</td>
<td>1% - 20%</td>
</tr>
<tr>
<td>Out-degree</td>
<td>15</td>
<td>4 - 18</td>
</tr>
<tr>
<td>Weighted out-degree (Total interbank liabilities)</td>
<td>5.8</td>
<td>3 - 13</td>
</tr>
<tr>
<td></td>
<td>34</td>
<td>180 - 12 000</td>
</tr>
</tbody>
</table>

Table 2: Nodes with highest systemic risk.
Credit Default swaps

- Credit default swaps are (off balance sheet) OTC contracts involving A selling protection to B on default of C.
- Upon default of C, A has to pay to B the loss given default, proportional to the notional of the CDS contract.
- June 2008: total interbank assets totaled $\approx 39$ trillion USD in June 2008
  Notional amount of single name credit default swaps $= 38$ trillion USD.
- If B already has exposure to C then the CDS has the effect of replacing the exposure $L_{BC}$ by an equivalent exposure $L_{BA}$ upon default of $C$. This modifies the network topology upon default of C but does not increase the number of links.
• In the case of *speculative* CDS i.e. when B has no exposure to C, default of C then has the effect of *triggering* a large exposure of B to A: a **new link** with large weight appears in the network. Typically C may be “distant” from A and B in the network.

• In the network terminology, they can be seen as contingent long-range links/shortcuts which appear in the graph when a default occurs.

• Adding a *small* proportion of CDS contracts in the networks can drastically change the topology of the network.

• Once the CDS are triggered the network behaves like a “small world”.
Figure 20: Default of a firm on which a lot of CDS protection has been sold can strongly affect exposures across the network. Blue: counterparty relations. Red: counterparty CDS exposures resulting from the default of a large name.
Figure 21: Increase in exposure sizes due to CDS triggered upon default of a large name.
Systemic impact of Credit Default swaps

- Simulation experiment: introduce a network of CDS contracts on top of an existing network of liabilities/exposures.
- total CDS notional = 20% of balance sheet sizes
- 50% of CDS are speculative.
- Protection selling is limited to ‘large’ institutions (e.g. 100 largest in balance sheet size)
- Underlyings of CDS are ‘large’ institutions (index names)
- If i has sold protection to j on k for a notional $N_{ij}$ then, upon default of k, i pays to j $N_{ij}(1 - R)$, absorbing a loss: $c_i \rightarrow c_i - N_{ij}(1 - R)$
  If $c_i < N_{ij}(1 - R)$, the protection seller defaults.
Figure 22: Effect of CDS on probability density of default impact (kernel estimator): total CDS notional = 20% of balance sheet sizes, 50% of CDS are speculative.
Figure 23: Effect of CDS on systemic risk index: total CDS notional = 20% of balance sheet sizes, 50% of CDS are speculative.
Central counterparties and CDS clearinghouses

- Central counterparties (CCP) have been proposed as a possible solution to counterparty risk and systemic risk management in CDS and other OTC markets.

- Replace bilateral CDS trades between counterparties by two symmetric trades between CCP and each counterparty.

- Insulates counterparties from each others default: mitigation of counterparty risk, a major concern since 2008

- By centralizing information and supervision can facilitate supervision and transparency.

- Mitigates moral hazard: intervention for “bailing out” a CPP is less problematic than bailing out individual banks

- Does a central counterparty reduce systemic risk?
Effect of a CDS clearinghouse

The effect of a central counterparty can be modeled by adding a node to the CDS network and redirecting all CDS contracts into this node.

For the central counterparty, the role of the capital buffer is played by margin deposits + a “Guarantee fund”. Each clearing participant contributes to a Guarantee fund.

The role of this fund is to reduce systemic risk by insulating clearing participants from the risk of the default of another clearing participant.

In accordance with BIS recommendations, the Guarantee Fund should cover losses associated with the simultaneous default of (a certain number of) largest clearing members in the event of deteriorating market conditions.
Figure 24: The Clearinghouse effect: Impact of a central counter-party on systemic risk index of financial institutions.
Figure 25: The Clearinghouse effect: Impact of a central counter-party on the distribution of systemic risk index of financial institutions.
Implementation and regulatory implications

- In some countries (Austria, Brazil,..) this data is already available to regulators and our methodology can be implemented at the level of the national regulatory body.

- In all countries banks and various financial institutions are required already to report risk measures (VaR, etc.) on a periodical basis to the regulator.

- Our approach would require these risk figures to be a disaggregated across large counterparties: banks would report a VaR/risk figure for exposures to each large counterparty.

- Many large investment banks already compute such exposures on a regular basis so requiring them to be reported is not likely to cause a major technological obstacle.
Conclusions

- The systemic risk impact of the failure of an institution may have little correlation with size or conventional risk measures of its portfolio. It depends on the network properties: centrality, connectivity, magnitude of exposures and liabilities,...

- We have proposed two measures of systemic risk impact of the default of a financial institution taking into account
  1. its connectivity with other market participants and the magnitude of its exposures: the Default Impact $DI(i)$
  2. the above + allowing for correlated market shocks across institutions during a crisis: the Systemic Risk Index $S(i)$.

- These criteria may be used as a tool in surveillance of systemic risk and for macro-prudential regulation.
• Actual capital, via the leverage coefficient, not risk-weighted capital seems to be a key determinant systemic risk.

• Imposing a cap on leverage is an effective mechanism for reducing default contagion and the probability of large systemic losses.

• Too big to fail/ Size is not the right criterion for determining systemic importance in absence of a maximal leverage ratio.

• Nodes with high systemic impact seem to be those who are major *counterparties* to large and well-connected nodes.

• Introduction of credit default swaps can increase default impact and systemic risk impact of large institutions

• Presence of a large notional volume of speculative credit default swap can distort the relation between systemic risk impact and firm properties (size, connectivity).
• The network approach allows to analyze the systemic impact of credit default swaps. In particular it illustrates that credit default swaps introduce *contingent* long-range links between institutions that can increase the range of contagion.

• Implementation will require reporting of large counterparty and CDS exposures to the regulator.

• Systemic risk involves understanding structure and dynamics of complex financial networks. Efficient methods for *large scale simulation* of realistic network models provide better insight than equilibrium models based on simplistic/homogeneous network structures.
References